La Paz Community School DATE:

Math SL – 11th Grade

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1. Let $f(x) = 2x^2 + x - 4,\ x \in \mathbb{R}$ and $g(x) = 2x - 1,\ x \in \mathbb{R}$

a) Find $f(-2)$

b) Solve the equation $f(x) = -1$

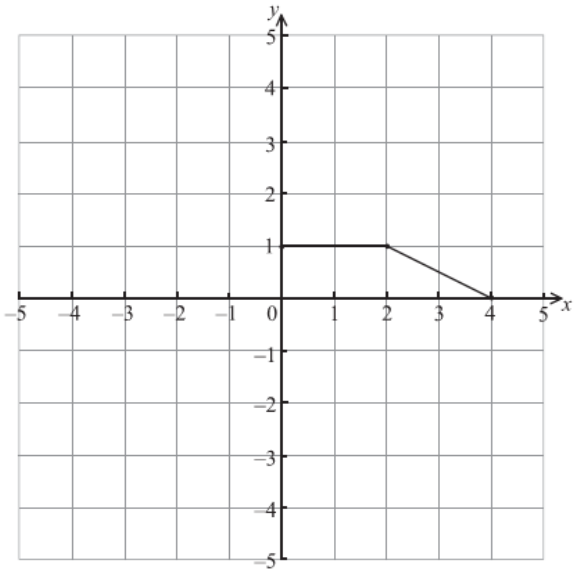
c) Find $g(-1)$

d) Given that $g(a) = g(2a-1)$, find the value of a.

e) Write down the vertex of f

f) Find the coordinates of the point of intersection of $f(x)$ and $g(x)$

2. Consider the graph of the function $y = f(x)$, for $0 \le x \le 4$.



a) Find $f(2)$

b) Estimate the solution of the equation $f(x) = 0.5$

c) Graph and label clearly $g(x) = f(x-2) + 1$

d) Graph and label clearly $h(x) = 2f(x)$

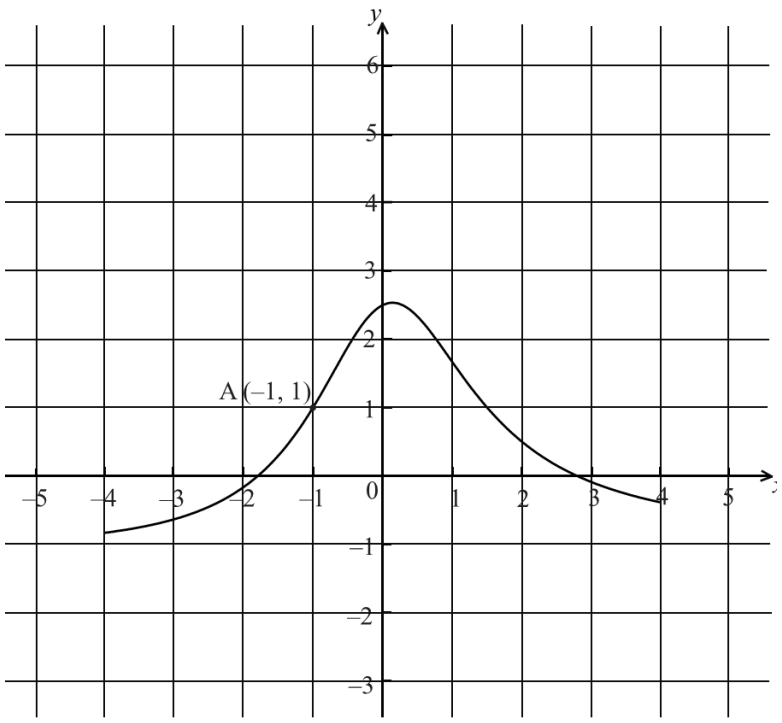
3. Let $f(x)=\sqrt{x+2},\ x\geq -2$ and $g(x)=x^2, x\in\mathbb{R}$.

a) Find $(g\circ f)(3)$

b) Find the range of $f(x)$

c) Find $f^{-1}(x)$ and state its domain and its range.

4. The graph of a function f is shown in the diagram below. The point $A(-1,1)$, in on the graph and $y=-1$ is a horizontal asymptote.



a) Estimate the value of $f(1.5)$

b) Estimate the solutions of $f(x)=1$

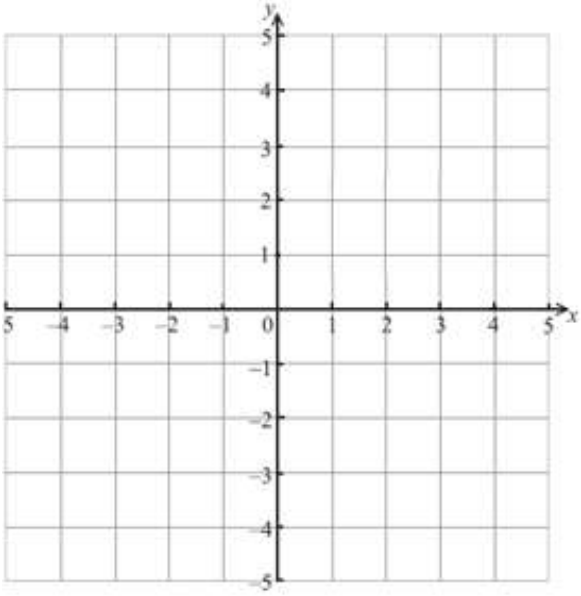
c) On the diagram, sketch the graph of $g(x)=f(x+1)+3$.

d) Write down the horizontal asymptote of g

e) Let A' be the point on the graph of g corresponding to point A . Write down the coordinates of A'

5. Let $g(x)=2x+1,\ f(x)=\dfrac{3x}{x-2},\ x\neq 2$

a) Sketch the graph of both functions in the following diagram



b) Find the coordinates of the points of intersection of $f(x)$ and $g(x)$

c) Find an expression for $(h\circ g)(x)$

d) Solve the equation $(h\circ g)(x)=0$

6. The function f is given by $f(x)=ax^3+bx^2+cx+d$, where a,b,c and d are integers.

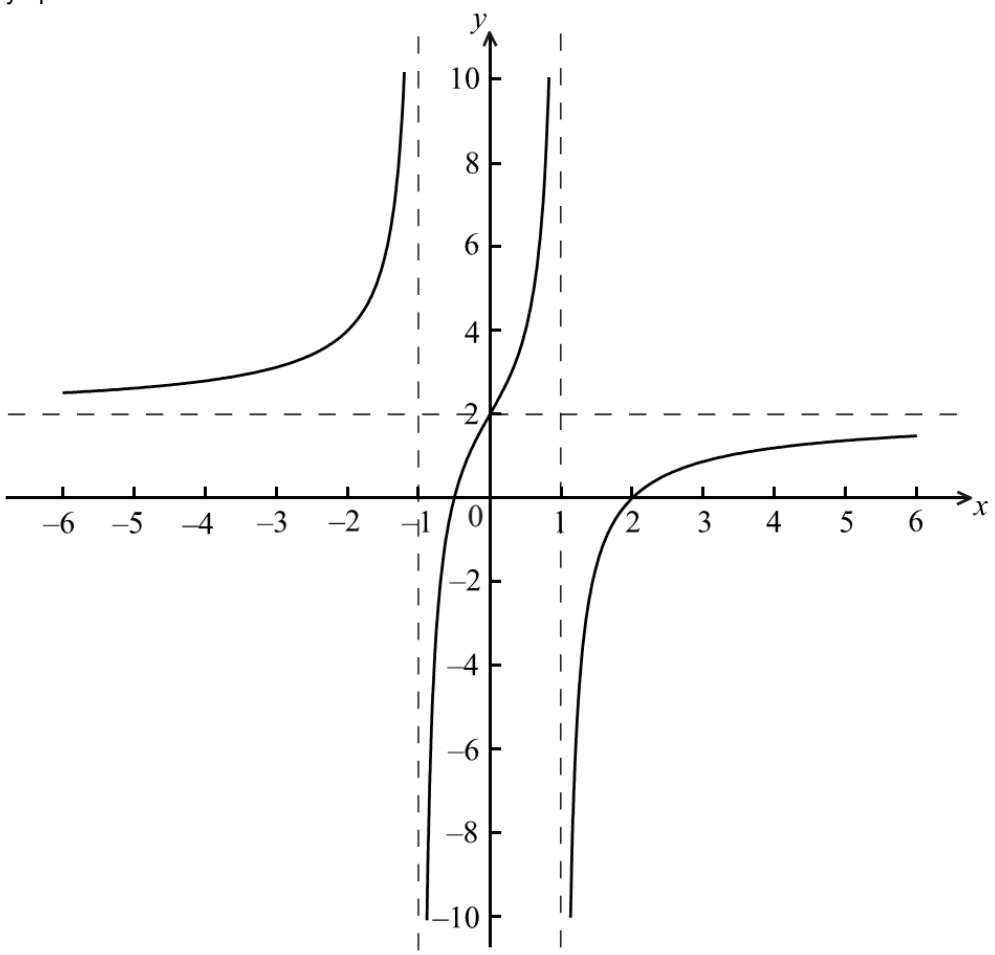
a) The graph of $f(x)$ passes through the origin. Find d

b) The graph of $f(x)$ intersects the x-axis also at $x=-1$. Show that $b=a+c$

c) The graph of $f(x)$ passes also through $(1,4)$ and $(2,30)$. State a system of equations to find a and c Solve that system and then write explicitly the rule for f

d) That rule can be written in the form $x(x+r)(px+q)$ where r,p and q are integers. Find them.

7. Let $f(x) = a - \dfrac{3x}{x^2 - b}$ where a and b are positive real numbers. Part of the graph of $f(x)$, including its asymptotes is shown below



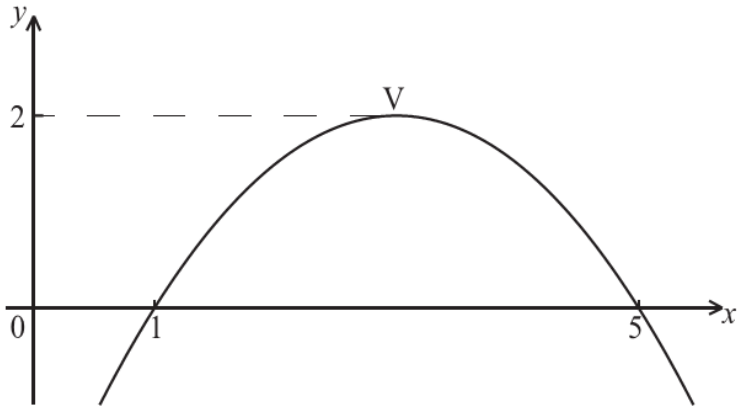
e) The equations of the asymptotes are $x = 1, x = -1, y = 2$. Write down the values of a and b

f) Find the intercepts with the axis.

g) Sketch the graph of the line l with slope -2 that intercept the y axis at $(0, 2)$

h) How many times the line l intersects the graph of $f(x)$?

8. Part of the graph of the function $f(x)=a(x-p)^2+q$ is given in the diagram below. The x intercept are $(1,0)$ and $(5,0)$. The vertex is $V(p,2)$



a) Write down the value of p

b) Write down the value of q

c) Find a

d) The function $g(x)$ is linear, passes through $(1,0)$ and $(4,3)$. Find the rule $g(x)$.

e) Find the coordinates of the points of intersection of $f(x)$ with the function $g(x)$

9. Consider the two different quadratic function of the form $f(x)=9x^2-qx+16$ that has its vertex on the x-axis.

a) Find both values of q

b) For the greater value of q, solve $f(x)=0$

c) Find the coordinates of the point of intersection of the two graphs

10. The monthly profit P, in thousands of dollars, of a company can be estimated by the formula $P(x)=-3x^2+30+12$, where x is the number of units sold per month.

a) Find the number of units that must be sold by the company to maximize its profit and then find the maximum profit.

b) Find that maximum profit

11. Given that $n = \log_a 2$, $m = \log_a 5$, express the following in terms of n and / or m

a) $\log_a 10$

b) $\log_a 32$

c) $\log_a \frac{1}{4}$

d) $\log_a 625$

e) $\log_a \sqrt[3]{5}$

f) $\log_a 200$

g) $\log_a \frac{5}{2}$

h) $\log_a \frac{125}{4}$

i) $\log_5 2$

j) $\log_2 25$

12. Let $\ln a = n,\ \ln b = q$, write the following expression in terms of p and / or q

a) $\ln a^3 b^5$

b) $\ln \frac{\sqrt{b}}{a^2}$

c) $\log_a b$

d) $\log_b a^3$

13. The expression $\left(3^x\right)^2 + \left(3^x\right) - 12$ can be written in the form $\left(3^x + a\right)\left(3^x + b\right)$ where $a, b \in \mathbb{Z}$

a) Find the value of a and b

b) Hence find the exact solution of the equation $\left(3^x\right)^2 + \left(3^x\right) - 12 = 0$ and explain why there is only one solution.

14. The function $f(x)$ and $g(x)$ are defined by $f(x)=10^x$ and $g(x)=\log(1-x)$

a) State the maximum domain of $g(x)$

b) Write down $f^{-1}(x)$

c) Write down $g^{-1}(x)$

d) Find $f \circ g$ and its inverse function

e) Find $g \circ f$ and its inverse function

15. The population of Guanacaste at the end of 1980 was $800\,000$. The population increases by 1.4% per year.

a) Write down the population at the end of 1981

b) Let x to be the number of years after 1980. Explain why the population at the year $1980+x$ should be $f(x)=800\,000(1,014)^x$

c) Find the population in 2016

d) When the population will be $2500\,000$?

16. Let $f(x)=\ln(x+4),\ x>-4$ and $g(x)=e^{x-2}, x>0$

a) Write down the x-intercept of the graph of $f(x)$

b) Write down the $f(-3.99)$

c) Find the range of f

d) Find the range of g

e) Find the coordinates of the point of intersection of $f(x)$ and $g(x)$

17. Find the exact value of x in each of the following equation

e) $3^{2x+1}=81$

f) $\log_a(2x+1)=2$

Mathematics SL formula booklet

1.2	Exponents and logarithms	$a^x = b \quad \Leftrightarrow \quad x = \log_a b$
	Laws of logarithms	$\log_c a + \log_c b = \log_c ab$ $\log_c a - \log_c b = \log_c \frac{a}{b}$ $\log_c a^r = r \log_c a$
	Change of base	$\log_b a = \frac{\log_c a}{\log_c b}$

2.4	Axis of symmetry of graph of a quadratic function	$f(x) = ax^2 + bx + c \quad \Rightarrow$ axis of symmetry $x = -\frac{b}{2a}$
2.6	Relationships between logarithmic and exponential functions	$a^x = \mathrm{e}^{x \ln a}$ $\log_a a^x = x = a^{\log_a x}$
2.7	Solutions of a quadratic equation	$ax^2 + bx + c = 0 \quad \Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \ a \neq 0$
	Discriminant	$\Delta = b^2 - 4ac$